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ON THE LIQUID FILM OF NUCLEATE BOILING

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Abstract—By examining the motion of liquid surrounding a bubble close to the heating surface accompanying bubble ebullition and growth, it is argued for a liquid thin layer to be formed fluid-dynamically or thermally at the base of the bubble on the heating surface during nucleate boiling, of the thickness inversely proportional to the square root of Reynolds number based on the bubble radius and its growth rate or to the bubble radius times the density ratio of vapor to liquid.

In the early stage of bubble growth, the liquid film is influential on the process only through the associated increase in liquid–vapor interfacial area. In the advanced stage, a bubble with liquid film grows at the rate similar to that without film, though the superheat of liquid has less influence upon the former. With a highly thinner film, the growth rate is affected by thermal properties of the heater, being too high to be realized in usual nucleate boiling of water.

NOMENCLATURE

c, specific heat of constant pressure;
L, latent heat of evaporation of liquid;
p, pressure;
q, heat flow;
r, co-ordinate parallel to heating surface;
R, radius of liquid–vapor interface of bubble;
R_0, radius of bubble;
S, area of liquid–vapor interface of bubble;
t, time;
T, temperature;
T_H, maximum temperature of heating material;
T_L, maximum temperature of surrounding liquid;
T_S, saturation temperature of liquid;
u, v, components of velocity of liquid (Fig. 1);
V, volume of bubble;
z, co-ordinate perpendicular to heating surface.

Greek symbols
α, thermal diffusivity;
δ, thickness of liquid layer;
λ, thermal conductivity;
ν, kinematic viscosity;
ρ, density;
φ, contact angle of bubble.

Subscripts
a, liquid–vapor interface except that of liquid film;
b, liquid–vapor interface of liquid film;
g, vapor in bubble;
h, heating material;
l, liquid.

INTRODUCTION

THE HIGH heat transfer rates associated with nucleate boiling processes have been attributed to the agitation of surrounding liquid by bubble formation and its departure from the heating surface, the heat flux being transferred once to the surrounding liquid before absorbed into bubbles. On the contrary, recent investigations have provided evidence that an evaporating liquid thin layer is formed at the base of bubbles during nucleate boiling so that the vapor inside the bubble cannot be always contacted with the heating surface, acting as the major heat transfer mechanism in nucleate boiling process [1–6].

Mesler *et al.* [1, 2] observed local surface temperature fluctuations of the heating surface by using a high response thermocouple and showed rapid cooling periods of short duration immediately after the ebullition of new bubbles followed by gradual reattainment of the initial temperature. Sharp [3] demonstrated the existence of a liquid film and measured its thickness profile by optical techniques of interference of monochromatic light and reflection of polarized light, to account for the existence of an evaporating thin layer inside the bubble on the heating surface. Torikai *et al.* [4] reported photographic observations of nucleate boiling process on the surface of a heated glass plate to demonstrate the presence of an evaporating thin layer. Cooper *et al.* [5] measured temperature fluctuations at several points on the surface of a heated glass plate by using small resistance thermometers consisting of thin films of metal and semiconductor, of which results lend support to the liquid film theory of boiling heat transfer. Katto *et al.* [5] investigated photographically the contribution of the liquid thin layer to the growth rate of bubbles between two flat plates, which did not necessarily correspond to the bubbles in nucleate boiling process.

If there exists such an evaporating liquid layer at the base of bubbles during nucleate boiling, the heat flux from the heating surface can be directly transferred into the vapor of bubble through the evaporation of the liquid film so that properties of the layer might control the major process of nucleate boiling.

The present study is to examine the possibility of the existence of an evaporating liquid thin layer at the base of bubbles during nucleate boiling and to discuss the effect of liquid film upon the mechanism of nucleate boiling heat transfer.

1. EXISTENCE OF LIQUID FILM AT THE BASE OF BUBBLES

If a liquid thin layer is to be built up at the base of bubbles on the heating surface during nucleate boiling, the process of the layer formation should be controlled by the motion of surrounding liquid associated with bubble growth and departure. First, we examine from the fluid-dynamical point of view the possibility of the formation of such a liquid thin layer and of its existence through the process of bubble growth. In addition to such a fluid-dynamical condition, the existence of the layer through the period of bubble growth may be dominated by the thermal process of evaporation. Secondly, we discuss the liquid film at the base of bubbles under such a thermally dominated process.

1. *Fluid-dynamically dominated layer*

The growth of a bubble generated on the heating surface makes its surrounding liquid act into motion to move in parallel with the heating surface in the vicinity of it. Let r and z be co-ordinates parallel and perpendicular to the heating surface, respectively (Fig. 1). It will be assumed that the associated liquid motion possesses cylindrical symmetry with respect to the z-axis and that the spacial variation of properties is much smaller in the r-direction than in the z-direction to be neglected.

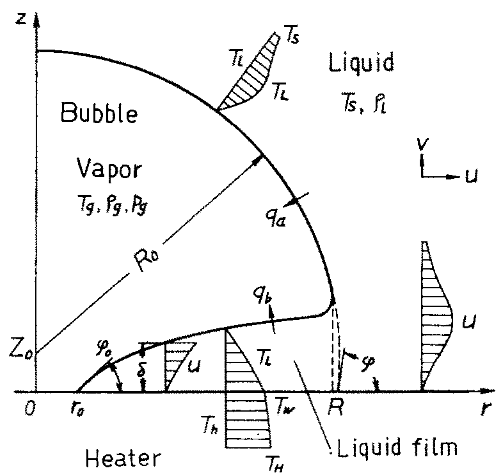

FIG. 1. Bubble with liquid film.

Then, the equations of continuity and motion with respect to the surrounding liquid become

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial v}{\partial z}=0 \tag{1.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=-\frac{1}{\rho_l}\frac{\mathrm{d}p}{\mathrm{d}r}+\nu_l\frac{\partial^2 u}{\partial z^2} \tag{1.2}$$

where ρ and ν are the density and the kinematic viscosity of liquid, respectively and are assumed to be constant.

We consider a liquid layer of thickness δ, in which equation (1.2) is valid, in the two regions of surrounding liquid; that is, the region inside the bubble ($0 \leqslant r \leqslant R$) and the region of outer part of bubble ($R \leqslant r$). The thickness δ in the former region means the thickness of a liquid film at the base of bubbles on the heating surface and that in the latter the thickness of boundary layer of liquid motion close to the heating surface. The position of the boundary of two regions, say R, can be assumed to be roughly equal to the projected point of bubble radius onto the heating surface, say R_0. At the liquid–vapor interface in the inside region, the heat flux from the heating surface through the layer makes liquid evaporate with the relation,

$$\rho_l L\left[v-u\frac{\partial\delta}{\partial r}-\frac{\partial\delta}{\partial t}\right]_\delta=\left[-\left(\lambda\frac{\partial T}{\partial z}\right)_l+\left(\lambda\frac{\partial T}{\partial z}\right)_g\right]_\delta\approx\left[-\left(\lambda\frac{\partial T}{\partial z}\right)_l\right]_\delta, \tag{1.3}$$

where L and λ are the latent heat of evaporation of liquid and the thermal conductivity, respectively. In addition to this, the continuity of momentum fluxes at the liquid–vapor interface gives the relationships of pressures and shearing stresses of liquid and vapor, though they are not necessary explicitly in the following discussion.

Let the co-ordinate z and the velocity components (u, v) be nondimensionalized by using the thickness of the layer $\delta(r, t)$ and the moving speed of the boundary of two layer-regions $\dot{R}(t)\ (\approx \dot{R}_0)$;

$$z=\delta y,\qquad u=\dot{R}u^*,\qquad v=\dot{R}u^*. \tag{1.4}$$

In the nondimensionalized system, equations (1.1)–(1.3) become

$$\frac{1}{r}\frac{\partial ru^*}{\partial r}-\frac{\delta'}{\delta}y\frac{\partial u^*}{\partial y}+\frac{1}{\delta}\frac{\partial v^*}{\partial y}=0 \tag{1.5}$$

$$\frac{R}{\dot{R}^2}\frac{\partial u}{\partial t}-\frac{R}{\dot{R}}\frac{\dot{\delta}}{\delta}y\frac{\partial u^*}{\partial y}+\frac{R}{r}\left[\frac{\partial ru^{*2}}{\partial r}-\frac{\delta'}{\delta}y\frac{\partial ru^{*2}}{\partial y}\right]+\frac{R}{\delta}\frac{\partial u^*v^*}{\partial y}=-\frac{R}{\rho_l\dot{R}^2}\frac{\mathrm{d}p}{\mathrm{d}r}+\left(\frac{R}{\delta}\right)^2\frac{\nu_l}{R\dot{R}}\frac{\partial^2 u^*}{\partial y^2} \tag{1.6}$$

$$v_1^*-\delta' u_1^*-\dot{\delta}=-\frac{\alpha_l}{\delta}H\left(\frac{\partial T^*}{\partial y}\right)_1, \tag{1.7}$$

where

$$\dot{R}=\frac{\mathrm{d}R}{\mathrm{d}t},\qquad \dot{\delta}=\frac{\partial\delta}{\partial t},\qquad \delta'=\frac{\partial\delta}{\partial r},$$

$$u_1^*=(u^*)_{y=1}$$

$$H=\frac{c_l(T_H-T_S)}{L}\qquad T=(T_H-T_S)\,T^*, \tag{1.8}$$

and T_H and T_S are the maximum temperature of the heating material and the saturation temperature of liquid, respectively.

Integration of equation (1.6) with respect to y from 0 to 1 and using equation (1.5) yields

$$\frac{\delta\dot{\delta}}{R^2\dot{R}}\int_0^1(u^*-u_1^*)\,\mathrm{d}y+\frac{\delta\delta'}{R^2}\int_0^1 u^*(u^*-u_1^*)\,\mathrm{d}y+\frac{\delta^2}{R^2}\left[\int_0^1\frac{1}{\dot{R}^2}\frac{\partial u}{\partial t}\mathrm{d}y+\frac{1}{r}\int_0^1\left\{\frac{\partial}{\partial r}(ru^{*2})\,\mathrm{d}y-u_1^*\frac{\partial}{\partial r}(ru^*)\right\}\mathrm{d}y+\frac{1}{\rho_l\dot{R}^2}\frac{\mathrm{d}p}{\mathrm{d}r}\right]=\frac{\nu_l}{R^2\dot{R}}\left[\left(\frac{\partial u^*}{\partial y}\right)_1-\left(\frac{\partial u^*}{\partial y}\right)_0\right]. \tag{1.9}$$

By the same manipulation to equation (1.7), we obtain

$$\frac{\delta\dot{\delta}}{R^2\dot{R}} + \frac{\delta\delta'}{R^2}\int_0^1 u^*\,\mathrm{d}y + \frac{\delta^2}{R^2}\int_0^1 \frac{1}{r}\frac{\partial}{\partial r}(ru^*)\,\mathrm{d}y = \frac{\nu_l}{R^2\dot{R}}\frac{\alpha_l}{\nu_l}H\left(\frac{\partial T^*}{\partial y}\right)_1. \qquad (1.10)$$

Elimination of $\dot{\delta}$ from equation (1.9) and (1.10) leads to

$$a_m\left(\frac{\delta^2}{R^2}\right)' + b_m\left(\frac{\delta^2}{R^2}\right) = c_m\frac{\nu_l}{R\dot{R}}, \qquad (1.11)$$

$$a_m = \tfrac{1}{2}[(\int_0^1 u^*\,\mathrm{d}y)^2 - \int_0^1 (u^*)^2\,\mathrm{d}y]$$

$$b_m = \frac{1}{r}\left[\int_0^1 \frac{\partial}{\partial r}(ru^*)\,\mathrm{d}y\int_0^1 u^*\,\mathrm{d}y - \int_0^1 \frac{\partial}{\partial r}(ru^{*2})\,\mathrm{d}y\right] - \int_0^1\left[\frac{R}{\dot{R}^2}\frac{\partial u}{\partial t} + \frac{R}{\rho_l\dot{R}^2}\frac{\mathrm{d}p}{\mathrm{d}r}\right]\mathrm{d}y$$

$$c_m = \frac{1}{R}\left[\left(\frac{\partial u^*}{\partial y}\right)_0 - \left(\frac{\partial u^*}{\partial y}\right)_1 + H\frac{\alpha_l}{\nu_l}\left(\frac{\partial T^*}{\partial y}\right)_1 \times\int_0^1 (u^* - u_1^*)\,\mathrm{d}y\right] \approx \frac{1}{R}\left[\left(\frac{\partial u^*}{\partial y}\right)_0 - \left(\frac{\partial u^*}{\partial y}\right)_1\right].$$

Since the equation of motion of liquid is a differential equation of the first order with respect to r, the equation for δ demands only one boundary condition with respect to r. With the boundary condition of $\delta = 0$ at $r = r_0$, the formal solution of equation (1.11) gives the thickness profile $\delta(r)$ as

$$\left(\frac{\delta}{R}\right)^2 = \frac{\nu_l}{R\dot{R}}\exp\left(-\int_{r_0}^{r}\frac{b_m}{a_m}\mathrm{d}r\right)\int_{r_0}^{r}\frac{c_m}{a_m} \times\exp\left(\int_{r_0}^{r}\frac{b_m}{a_m}\mathrm{d}r\right)\mathrm{d}r. \qquad (1.12)$$

At the point r_0 where $\delta = 0$ and $u = 0$, liquid and vapor are in a hydrostatical equilibrium so that contact angle of liquid–vapor interface against the heating surface can be kept constant φ_0. Accordingly, the heat balance at $r = r_0$ provides the relation between the heat flux at $r = r_0$, say $\hat{q}_0$, and the change of r_0 in time,

$$\hat{q}_0 = \tfrac{1}{2}\rho_l L\tan\varphi_0\frac{\mathrm{d}r_0}{\mathrm{d}t}.$$

From the above relation with the initial condition of $r_0 = r_{00}$ at time origin $t = 0$, we obtain

$$r_0 = r_{00} + \frac{2}{\rho_l L\tan\varphi_0}\int_0^t \hat{q}_0\,\mathrm{d}t. \qquad (1.13)$$

If we use the area-mean of heat flux as $\hat{q}_0$, the heat flux may be roughly estimated as $\hat{q}_0 \approx \rho_g L\dot{R}_0$, so that the second term of equation (1.13) is of the order of $(\rho_g/\rho_l)R$ to be neglected compared with R. Thus, it leads to $r_0 < R$. This means, from equation (1.12), that the liquid thin layer can always exist at the base of bubbles on the heating surface so long as $a_m c_m > 0$, without regard to the value of b_m, and that the thickness of the layer can be expressed as

$$\frac{\delta}{R} \sim \sqrt{\left(\frac{\nu_l}{R\dot{R}}\right)}. \qquad (1.14)$$

Rewriting equation (1.14) as

$$(\rho_l\dot{R}^2)\,\pi\delta R \sim \rho_l\nu_l\frac{\dot{R}}{\delta}\pi R^2$$

tells evidently its meaning that the momentum flux of liquid in the film should be balanced instantaneously with the viscous stress acting on the heating surface. When the bubble radius is proportional to the square root of time and it can be assumed that $R \approx R_0$, we obtain the relation that $R/\dot{R} = 2t$, with which equation (1.14) becomes

$$\delta \sim \sqrt{(\nu_l t)}. \qquad (1.15)$$

In the case of saturated water of $\nu_l = 0{\cdot}002\ \mathrm{cm^2/s}$, the thickness of the film at $t = 2$ ms is thus estimated to be about 10 μm.

As to the signs of a_m and c_m, the following consideration leads to the relation $a_m c_m > 0$. It can be assumed for the liquid motion in the film that $(\partial u^*/\partial y)_1 > (\partial u^*/\partial y)_0$, that is, $c_m < 0$ and that $\partial^n u^*/\partial y^{*n} \geqslant 0$ ($n = 1, 2, 3, \dots$). If the velocity profile in the film is expressed by a polynomial function of

$$u^* = \sum_{i=1}^{n} a_i y^i \qquad (a_i \geqslant 0),$$

a_m is then given by

$$a_m = \frac{1}{2}\sum_{i,j}^{n} a_i a_j \left(\frac{1}{i+1}\frac{1}{j+1} - \frac{1}{i+j+1}\right) < 0.$$

Though the contribution of the film to the bubble growth rate provides a relation between R and δ, we have no further consideration about the fluid-dynamically dominated liquid film. The fluid-dynamical behavior of the liquid film can be characterized conspicuously in the early stage of bubble growth when the liquid in the layer is of intense motion. As shown later, however, the layer is not considered to have great influence upon the growth process of bubble in its early stage.

2. *Thermally dominated layer*

Next, we consider the existence of liquid thin layer under the thermally dominated condition such that the evaporation of liquid at the interface controls the process. It can be assumed that the variation in temperature of liquid in the vicinity of the heating surface is much smaller in the r-direction than in the z-direction. The energy equation is then

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + v\frac{\partial T}{\partial z} = \alpha_l \frac{\partial^2 T}{\partial z^2}. \qquad (1.16)$$

Equation (1.16) with equations (1.4) and (1.8) becomes

$$\frac{R}{\dot{R}}\frac{\partial T^*}{\partial t} - \frac{R\dot{\delta}}{\dot{R}\delta} y \frac{\partial T^*}{\partial y} + \frac{R}{r}\left[\frac{\partial(ru^*T^*)}{\partial r} - \frac{\delta'}{\delta} y \frac{\partial(ru^*T^*)}{\partial y}\right] + \frac{R}{\delta}\frac{\partial(v^*T^*)}{\partial y} = \frac{R^2}{\delta^2}\frac{\alpha_l}{R\dot{R}}\frac{\partial^2 T^*}{\partial y^2}. \qquad (1.17)$$

Integration of equation (1.17) with respect to y from 0 to 1 using equation (1.5) gives

$$\frac{\delta\dot{\delta}}{R^2\dot{R}}\int_0^1 (T^* - T_1^*)\,dy + \frac{\delta\delta'}{R^2}\int_0^1 u^*(T^* - T_1^*)\,dy + \frac{\delta^2}{R^2}\left[\int_0^1 \frac{1}{\dot{R}}\frac{\partial T^*}{\partial t}\,dy + \frac{1}{r}\int_0^1\left\{\frac{\partial}{\partial r}(ru^*T^*) - T_1^*\frac{\partial}{\partial r}(ru^*)\right\}dy\right] = \frac{\alpha_l}{R^2\dot{R}}\left[\left(\frac{\partial T^*}{\partial y}\right)_1 - \left(\frac{\partial T^*}{\partial y}\right)_0\right]. \qquad (1.18)$$

Accordingly, equations (1.10) and (1.18) yield

$$a_e\left(\frac{\delta^2}{R^2}\right)' + b_e\left(\frac{\delta^2}{R^2}\right) = c_e\frac{\alpha_l}{R\dot{R}} \qquad (1.19)$$

where

$$a_e = \tfrac{1}{2}\left[\int_0^1 u^*\,dy \int_0^1 T^*\,dy - \int_0^1 u^*T^*\,dy\right]$$

$$b_e = \frac{1}{r}\left[\int_0^1 \frac{\partial}{\partial r}(ru^*)\,dy \int_0^1 T^*\,dy - \int_0^1 \frac{\partial}{\partial r}(ru^*T^*)\,dy\right] - \int_0^1 \frac{R}{\dot{R}^2}\frac{\partial T^*}{\partial t}\,dy$$

$$c_e = \frac{1}{R}\left[\left(\frac{\partial T^*}{\partial y}\right)_0 - \left(\frac{\partial T^*}{\partial y}\right)_1 + H\left(\frac{\partial T^*}{\partial y}\right)_1 \int_0^1 (T_1^* - T^*)\,dy\right].$$

The solution of equation (1.19) for $a_e \neq 0$ is

$$\left(\frac{\delta}{R}\right)^2 = \frac{\alpha_l}{R\dot{R}}\exp\left(-\int_{r_0}^{r}\frac{b_e}{a_e}\,dr\right)\int_{r_0}^{r}\frac{c_e}{a_e} \times \exp\left(\int_{r_0}^{r}\frac{b_e}{a_e}\,dr\right)dr. \qquad (1.20)$$

Since $r_0 < R$ as mentioned previously, equation (1.20) shows the existence of the liquid film at the base of bubbles as long as $a_e c_e > 0$, of which thickness is

$$\frac{\delta}{R} \sim \sqrt{\left(\frac{\alpha_l}{R\dot{R}}\right)}. \tag{1.21}$$

Concerning the signs of a_e and c_e, the plausible relations of temperature profile of the liquid in the layer that $(\partial T^*/\partial y)_1 < (\partial T^*/\partial y)_0$ and $(\partial^n T^*/\partial y^n) \leqslant 0$ $(n = 1, 2, 3, \ldots)$ lead to the result that $a_e > 0$, $c_e > 0$ and $a_e c_e > 0$, in the same way as that for velocity profile.

The thermally dominated layer, contrasted with the fluid-dynamically dominated one, may have much contribution to the bubble growth, so that R and δ have the following relation of evaporation. If we assume that R is nearly equal to R_0 and that the bubble growth is attributed mainly to the evaporation of liquid in the layer, we obtain the relation of bubble growth

$$\rho_g V^* \dot{R} = -\rho_l \alpha_l H \int_{S_b} \frac{1}{\delta}\left(\frac{\partial T^*}{\partial y}\right)_1 \mathrm{d}\left(\frac{S_b}{\pi R^2}\right), \tag{1.22}$$

where S_b is the area of liquid–vapor interface of the layer and V^* is a dimensionless quantity defined by equation (2.10). The temperature gradient at the liquid–vapor interface is given by equation (1.19) as

$$\left(\frac{\partial T^*}{\partial y}\right)_1 = \frac{1}{H}\left[\frac{R^2\dot{R}}{\alpha_l}\left\{a_e\left(\frac{\delta^2}{R^2}\right)' + b_e\frac{\delta^2}{R^2}\right\} + \left(\frac{\partial T^*}{\partial y}\right)_1 - \left(\frac{\partial T^*}{\partial y}\right)_0\right]\left[\int_0^1 (T_1^* - T^*)\,\mathrm{d}y\right]^{-1}. \tag{1.23}$$

Letting the maximum of δ be $\bar{\delta}$ and setting

$$\delta = \bar{\delta}\cdot\delta^*,$$

we obtain the relation of $\bar{\delta}$ from equation (1.22) as

$$\bar{a}_e\left(\frac{\bar{\delta}}{R}\right)^2 + \bar{b}_e\frac{\rho_g}{\rho_l}\frac{\bar{\delta}}{R} - \frac{\alpha_l}{R\dot{R}}\bar{c}_e = 0, \tag{1.24}$$

where

$$\bar{a}_e = \int_{S_b} R\left[a_e(\delta^{*2})' + b_e\delta^{*2}\right]\left[\delta^*\int_0^1 (T^* - T_1^*)\,\mathrm{d}y\right]^{-1}\mathrm{d}\left(\frac{S_b}{\pi R^2}\right)$$

$$\bar{b}_e = V^*$$

$$\bar{c}_e = \int_{S_b}\left[\left(\frac{\partial T^*}{\partial y}\right)_0 - \left(\frac{\partial T^*}{\partial y}\right)_1\right]\left[\delta^*\int_0^1 (T^* - T_1^*)\,\mathrm{d}y\right]^{-1}\mathrm{d}\left(\frac{S_b}{\pi R^2}\right).$$

Since $T^* > T_1^*$ and $(\partial Y^*/\partial y)_0 > (\partial T^*/\partial y)_1$, as mentioned above, we obtain that $\bar{c}_e > 0$. The value of $\bar{b}_e$ is always positive and $\bar{a}_e$ is of a small value in the case of less intense motion of liquid. Thus, it is known that equation (1.24) has always a positive root with respect to $(\bar{\delta}/R)$. This is valid for the case of $\bar{a}_e = 0$, that is, for no motion of liquid in the layer. The thickness of the layer from equations (1.21) and (1.24) is given by

$$\frac{\bar{\delta}}{R} \sim \frac{\rho_g}{\rho_l}. \tag{1.25}$$

Now, it is concluded that the liquid motion accompanying bubble growth and departure can be associated with the formation of a liquid thin layer at the base of bubbles on the heating surface, as long as the surface is wetted by liquid, and that such a liquid layer can also exist in the case of thermally dominated process of evaporation.

2. NUCLEATE BOILING WITH LIQUID FILM

Under the presence of liquid thin layer at the base of bubbles on the heating surface, we consider the mechanism of nucleate boiling process. For simplicity, we take the following assumptions; (1) the shape of bubbles is a part of the sphere of radius R_0 with the center Z_0 over the heating surface, (2) the vapor inside the bubble is of the saturated state at temperature

T_g and pressure p_g, (3) the effects of the acceleration of gravity and the viscosity except in the vicinity of the heating surface are neglected, and (4) the growth of bubbles is attributed to the evaporation of liquid both at the liquid film and at the other part of liquid–vapor interface.

The motion of liquid in the thin layer should be taken into consideration when the layer is examined about its existence under a fluid-dynamically controlled process such as in the initial period of bubble growth accompanying strong motion of liquid. However, through the almost whole period of bubble growth except in the initial stage, it can be assumed that (5) the motion of liquid in the layer is neglected.

The characteristic features of the mechanism of nucleate boiling with the presence of liquid film at the base of bubbles different from those without the film can be attributed to the assumption (4) of mass conservation. The equation of motion of liquid and of bubble are identical with those under no presence of liquid film [7]. We examine the mechanism of nucleate boiling under the existence of such a liquid film and its contribution to the boiling process. Hereafter, quantities of heating material, liquid and vapor are denoted by subscripts h, l and q, respectively.

From assumption (4), the heat flow q transferred into the bubble through the liquid–vapor interface can be expressed as the summation of the flow through the interface of the liquid film q_b and that through the other part of liquid–vapor interface of bubble q_a, that is,

$$q = q_a + q_b. \tag{2.1}$$

Denoting the interfacial surface areas of the liquid film and of the other part as S_b and S_a, respectively, we can express the flows as

$$\begin{aligned} q_b &= \int_{S_b} \left(-\lambda_l \frac{\partial T_l}{\partial z}\right)_\delta \mathrm{d}S_b, \\ q_a &= \int_{S_a} \left(-\lambda_l \frac{\partial T_l}{\partial R}\right)_{R_0} \mathrm{d}S_a. \end{aligned} \tag{2.2}$$

On the other hand, the energy relation of vapor inside the bubble of volume V gives

$$q = L\frac{\mathrm{d}}{\mathrm{d}t}(\rho_g V) + \rho_g V c_g \frac{\mathrm{d}T_g}{\mathrm{d}t} \approx L\frac{\mathrm{d}}{\mathrm{d}t}(\rho_g V), \tag{2.3}$$

where the change in the interfacial energy is neglected.

These heat flows expressed in equation (2.2) can be determined by temperature distributions of liquid inside the liquid film and in the vicinity of the other part of the interface. The temperature profile in the former region is obtainable by solving the temperature field in the film as well as in the heating material (Appendix), while that in the latter is given by the same way as used by Plesset or Forster.

To see how these characteristics are important, it is useful to bring into perspective the nature of the process in the initial stage of bubble growth ($\alpha_l t/\delta^2 \ll 1$) and in the final stage of it ($\alpha_l t/\delta^2 \gg 1$). According to Appendix, the heat flows can be given by

$$q_a = \begin{cases} \dfrac{\lambda_l}{\sqrt{(4\pi\alpha_l)}} S_a \displaystyle\int_0^t \frac{T_g - T_L}{(t-\tau)^{\frac{3}{2}}}\mathrm{d}\tau & \dfrac{\alpha_l t}{\delta^2} \ll 1 \\ \dfrac{\lambda_l}{\sqrt{(\pi\alpha_l t)}} S_a (T_L - T_g) & \dfrac{\alpha_l t}{\delta^2} \gg 1 \end{cases} \tag{2.4}$$

$$q_b = \begin{cases} \dfrac{\lambda_l}{\sqrt{(4\pi\alpha_l)}}\left[1 + \dfrac{\lambda_l}{\lambda_h}\sqrt{\left(\dfrac{\alpha_h}{\alpha_l}\right)}\right]^{-1} S_b \displaystyle\int_0^t \frac{T_g - T_H}{(t-\tau)^{\frac{3}{2}}}\mathrm{d}\tau & \dfrac{\alpha_l t}{\delta^2} \ll 1 \\ \displaystyle\int_{S_b} \frac{\lambda_l}{\delta}\left[1 + \frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_h t)}}{\delta}\right]^{-1} \mathrm{d}S_b (T_H - T_g) & \dfrac{\alpha_l t}{\delta^2} \gg 1 \end{cases} \tag{2.5}$$

where

$$(t-\tau)^{\frac{1}{2}} \equiv \lim_{z \to 0} (t-\tau)^{\frac{1}{2}} \exp\left(-\frac{z^2}{4\alpha_l(t-\tau)}\right)$$

and T_L and T_H are the maximum temperatures of liquid and of the heating material, respectively.

Since $T_L \approx T_H$ at the initial bubble growth and for usual heating materials $\lambda_l/\lambda_h\sqrt{(\alpha_h/\alpha_l)} \ll 1$, equations (2.4) and (2.5) give

$$\frac{q_b}{q_a} \approx \frac{S_b}{S_a}. \quad (2.6)$$

It is thus noted that the increment in the heat flow transferred into the bubble due to the liquid film can be regarded to be roughly equal to the amount of the associated increase in the interfacial area, so that the total heat flow can be given by

$$q = \frac{\lambda_l}{\sqrt{(4\pi\alpha_l)}}\left(1 + \frac{S_b}{S_a}\right)S_a \int_0^t \frac{T_g - T_H}{(t-\tau)^{\frac{1}{2}}}\,\mathrm{d}\tau. \quad (2.7)$$

Expanding the quantities of bubble shape R_0 and Z_0 and those of vapor inside the bubble ρ_g, p_g and T_g with respect to time t as

$$R_0 = R_e + R_1 t + R_2 t^2 + \ldots,$$
$$T_g = T_e + T_1 t + T_2 t^2 + \ldots, \text{etc.},$$

we can obtain T_n as a function of $(T_{n-1}, \ldots, T_e, R_n, \ldots, R_e, Z_n, \ldots, Z_e)$ from equations (2.3) and (2.7) with the state equation of ideal gas and the relation of saturation [assumption (2)], and R_n and Z_n as functions of $(T_n, \ldots, T_e, R_{n-1}, \ldots, R_e, Z_{n-1}, \ldots, Z_e)$ form equations of motion in the R_0- and z-directions with respect to the bubble [7]. The characteristic features of the process associated with the liquid film are the increment in the heat flow transferred into the bubble by the amount $(1 + S_b/S_a)$ and the modification of the equation of motion in the z-direction of liquid close to the heating surface. The latter may have less influence upon the resulted process of bubble growth because of less inertia of liquid in the vicinity of the heating surface compared with that of the whole liquid surrounding the bubble. Concerning the former, if we take the initial bubble growth rate $(\dot{R}_0)_{t=0} = R_1$ increased by $(1 + S_b/S_a)$, keeping the apparent contact angle of bubble constant, it can be easily seen that the process of bubble growth under the presence of the layer could be described in the same nondimensionalized expression as the process without the layer; in other words, the processes of bubble growth and departure from the heating surface under the existence of the film are identical with those without the film, only the rate of bubble growth being increased by the amount $(1 + S_b/S_a)$.

Next, we consider the process at the time close to the final stage of bubble growth. The ratio of the heat flows through interfaces is given by equations (2.4) and (2.5) as

$$\frac{q_b}{q_a} = \frac{\int_{S_b} \frac{\lambda_l}{\delta}\left[1 + \frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_h t)}}{\delta}\right]^{-1} \mathrm{d}S_b(T_H - T_g)}{\frac{\lambda_l}{\sqrt{(\pi\alpha_l t)}} S_a(T_L - T_g)}, \quad (2.8)$$

which means that $q_b \gg q_a$ in the final stage of bubble growth, so that

$$q \approx q_b \approx \int_{S_b} \frac{\lambda_l}{\delta}\left[1 + \frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_h t)}}{\delta}\right]^{-1} \mathrm{d}S_b(T_H - T_g). \quad (2.9)$$

For $\lambda_l/\lambda_h \cdot \sqrt{(\pi\alpha_h t)}/\delta \gg 1$, the total heat flow into the bubble is then given by

$$q \approx \frac{\lambda_h}{\sqrt{(\pi\alpha_h t)}} S_b(T_H - T_g). \quad (2.9)'$$

Representing the change in the mass of vapor inside the bubble in the form that

$$\frac{\mathrm{d}\rho_g V}{\mathrm{d}t} = \rho_g \pi R_0^2 \dot{R}_0 V^*, \quad (2.10)$$

we obtain, from equation (2.9)′,

$$\dot{R}_0 = \frac{1}{\rho_g L}\frac{\lambda_h}{\sqrt{(\pi\alpha_h t)}}\frac{S_b}{\pi R_0^2 V^*}(T_H - T_g). \quad (2.11)$$

When the apparent contact angle of bubble has little change in value with respect to time and the vapor temperature approaches the saturation temperature of liquid, equation (2.11) is reduced to

$$R_0 = \tfrac{1}{2}\Phi_b(\varphi)\frac{\lambda_h}{\lambda_l}\sqrt{\left(\frac{\alpha_l}{\alpha_h}\right)}\frac{2}{\sqrt{\pi}}\frac{\rho_l}{\rho_g}H\sqrt{(\alpha_l t)} \tag{2.12}$$

where

$$H = c_l(T_H - T_S)/L$$

and

$$\frac{2S_b}{\pi R_0^2 V^*} = \frac{2(1-\cos\varphi)}{(1+\cos\varphi)(2-\cos\varphi)} \equiv \Phi_b(\varphi).$$

For the process in which only q_a is taken into account, the same manipulation leads to the result

$$R_0 = \Phi_a(\varphi)\frac{2}{\sqrt{\pi}}\frac{\rho_l}{\rho_g}H\sqrt{(\alpha_l t)} \tag{2.13}$$

where

$$\Phi_a(\varphi) = \frac{2}{(1+\cos\varphi)(2-\cos\varphi)}.$$

For the nucleate boiling of saturated water, the values of Φ_a and Φ_b in equations (2.12) and (2.13) do not change considerably within the range of contact angle ($90° > \varphi > 60°$), being kept the order of unity ($\Phi_a \approx \Phi_b \approx 1$). Owing to equations (2.12) and (2.13), it is seen that the liquid film at the base of the bubble could make bubbles grow faster by the amount of about $\lambda_h/\lambda_l\sqrt{(\alpha_l/\alpha_h)}\,(\approx 10)$ than in the case without the film. $\Phi_a = \sqrt{3}$ by Plesset–Zwick's equation, while $\Phi_a = \pi/2$ for Forster–Zuber's result. With these values, equation (2.13) is observed accidentally to be consistent with the experimental results of bubble growth of nucleate boiling on a heating surface. Consequently, the bubble growth rate under the presence of the liquid film given by equation (2.12) may be about ten times higher than the value by equation (2.13) to be less realized in usual cases of saturated water.

It is possible to verify equation (2.12) by examining the effect of heating materials upon the process of bubble growth. The value of $\lambda_h/\sqrt{\alpha_h}$ for copper is about five times that for chromium, so that according to equation (2.12) bubbles on a heating surface of copper grow five times faster than those on a chromium heater. Such a material effect on the process, however, is attributed mainly to the difference of properties of heating materials very close to the heating surface. For lack of experimental information about the growth process accounting for such a material effect, further discussions on it must be abstained.

Next, we discuss the process at the intermediate period of bubble growth between the above two stages, in which the heat flow given by equation (2.9) can be approximated not by equation (2.9)′ but by

$$q = \int_{S_b} \lambda_l \frac{T_H - T_g}{\delta}\,\mathrm{d}S_b. \tag{2.14}$$

Since at the period it may be assumed that $R_0\dot{R}_0 \approx$ constant, equations (1.20) and (1.25) yield

$$\delta = A(r,t)\frac{\rho_g}{\rho_l}\sqrt{\left(\frac{r}{R_0}\right)}R_0, \tag{2.15}$$

where $A(r, t)$ is a weakly dependent function of r and t. Thus, equation (2.14) becomes

$$q = \int_0^{\sin\varphi} \lambda_l \frac{\rho_l}{\rho_g}\frac{T_H - T_g}{A}2\pi R_0\sqrt{\left(\frac{r}{R_0}\right)}\,\mathrm{d}\left(\frac{r}{R_0}\right) \approx \frac{4\pi}{3}(\sin\varphi)^{\frac{3}{2}}R_0\frac{\rho_l}{\rho_g}\frac{\lambda_l}{A}(T_H - T_S).$$

Solving equation (2.3) with the above heat flow leads to

$$R_0 = \sqrt{\left[\frac{\pi}{3}\Phi_{b'}(\varphi)\frac{1}{AH}\right]}\frac{2}{\sqrt{\pi}}\frac{\rho_l}{\rho_g}H\sqrt{\alpha_l t}, \tag{2.16}$$

where

$$\Phi_{b'}(\varphi) = \frac{2(\sin\varphi)^{\frac{1}{2}}}{(1+\cos\varphi)^2(2-\cos\varphi)}.$$

The value of A can be estimated according to the result of Katto's observation [6] of the growth process of two-dimension-like bubbles between flat plates, which was purely attributable to the evaporation of liquid film. For a two-dimension-like bubble growth, equation (1.24) is modified to

$$\frac{\delta}{R_0} \sim \frac{\rho_g}{\rho_l}\frac{h}{R_0}$$

where h is the height of the bubble, so that equation (2.15) can be reduced to

$$\delta \approx A\frac{\rho_g}{\rho_l}\sqrt{\left(\frac{r}{R_0}\right)}h. \quad (2.17)$$

Equation (2.14) with equation (2.17) leads to

$$\dot{R}_0 \approx \frac{4}{3AH}H^2\left(\frac{\rho_l}{\rho_g}\right)^2\frac{\alpha_l}{h^2}R_0. \quad (2.18)$$

From the typically observed result, $\dot{R}_0/R_0 = 0{\cdot}5\ \mathrm{ms}^{-1}$ for $T_H - T_S = 3°\mathrm{C}$ and $h = 0{\cdot}04$ cm, which leads to $A \approx 40$. Using this value of $A \approx 40$ and the value of $H\ (\approx 10^{-1} \sim 10^{-2})$ for usual nucleate boiling of saturated water, that is, $AH \approx 1$, which leads to $\sqrt{(\pi/3 \cdot \Phi_{b'}/AH)} \approx 1$, we can recognize equation (2.16) roughly consistent with the observed growth curves.

Upon comparing equation (2.16) with equation (2.13), one of the most conspicuous difference consists in the contribution of superheat $(T_H - T_S)$. According to equation (2.13) for the process without the liquid film, the bubble radius is proportional to the first power of superheat, while it grows in proportion to the square root of superheat in equation (2.16). Such a trend is always found regardless of the assumption of thickness profile of the liquid film, whenever the maximum of the film thickness is proportional to the bubble radius. It is, however, not easy to derive detailed information about the effect of superheat on the bubble growth of usual nucleate boiling within a narrow range of superheat. The bubble growth rate given by equation (2.13) may be overestimated because of the assumption of liquid temperature keeping the value of T_H through the whole range of bubble growth. The accordance of equation (2.13) with the experimental results may be attributed to the compensation for the overestimation by the contribution of the liquid film, that is, by equation (2.16).

Consequently, it can be noted, for usual nucleate boiling of saturated water, that the liquid film at the base of bubbles has not so great influence upon the bubble growth in its final stage as to show the material effect of the heater, and that in earlier stages it acts as making up for the decrease in the heat flux through the interface. It is not a so easily realized situation except at the time close to the final stage of bubble growth that the heat flux through the interface of the liquid film highly overwhelms that through the other part of interface.

CONCLUSION

By examining the motion of liquid surrounding a bubble in the vicinity of the heating surface accompanying bubble ebullition and growth, it is argued for a liquid thin layer to be formed fluid-dynamically at the base of the bubble on the heating surface during nucleate boiling. The thickness of the associated liquid film is inversely proportional to the square root of Reynolds number based on the bubble radius and its growth rate. In the process thermally controlled by evaporation of the film, it is proportional to the bubble radius times the density ratio of vapor and liquid.

At initial bubble growth, the liquid film is influential on the mechanism of nucleate boiling only through the associated increase in the liquid–vapor interface area of bubble; that is, it has no effect on the process of bubble growth expressed in dimensionless quantities based on the initial bubble growth rate increased by the amount of increment in the interfacial area of bubble. In advanced stages of bubble growth, the bubble radius is proportional to $\sqrt{(\alpha_l t)} \cdot \rho_l/\rho_g$ regardless of the presence of liquid film, though it is proportional to superheat $H[=c_l(T_H - T_S)/L]$ for bubbles without film and to $\sqrt{H}$ for those with film. At the time close to the final stage of bubble growth with further thinner films, the bubble radius becomes proportional to $\sqrt{(\alpha_l/\alpha_h)}$ $H \cdot \lambda_h/\lambda_l$ in which thermal properties of the heating material are participated.

By lack of experimental information about such an effect of superheat or heater material, it is difficult actually to confirm the characteristic influence of the liquid film upon the mechanism of nucleate boiling. For saturated water, the liquid film is not usually found to show the material effect of the heater upon bubble growth but seems to act as only a compensator for the decrease in the heat flux through the liquid–vapor interface except that of liquid film.

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APPENDIX

First, we consider the heat flow through the liquid film q_b. To obtain the temperature distribution in the liquid film, the temperature field in the heating material should be taken into account, because of appreciable changes in the latter, which are experimentally observed by several investigators. Since the heat flow parallel to the heating surface may be negligibly small compared with that perpendicular to the surface, the energy equations in the liquid film and in the heating material are

$$\frac{\partial T_h}{\partial t} = \alpha_h \frac{\partial^2 T_h}{\partial z^2}, \qquad \frac{\partial T_l}{\partial t} = \alpha_l \frac{\partial^2 T_l}{\partial z^2}, \tag{A.1}$$

where the liquid motion in the film is neglected with assumption. (5). Let the surface temperatures of the liquid film and of the heater be T_δ and T_w, respectively, and denote the temperature of the heating material far from the surface as T_H. The surface temperature of the film T_δ can be regarded roughly equal to that of vapor inside the bubble T_g. The heating material is sufficiently thick that T_H can remain constant in no response to the change in T_w.

Concerning the temperature of the heating material, introduction of

$$\theta_h = T_h - T_H \tag{A.2}$$

reduces equation (A.1) to

$$\frac{\partial \theta_h}{\partial t} = \alpha_h \frac{\partial^2 \theta_h}{\partial z^2} \tag{A.3}$$

with the boundary conditions

$$z = 0 \quad : \theta(0, t) = T_w(t) - T_H \equiv \theta_w(t)$$
$$z = -\infty : \theta(-\infty, t) = 0.$$

The solution of equation (A.3) is easily found to be

$$\theta_h(z, t) = \frac{2}{\sqrt{\pi}} \theta_w(0) \int_{-\infty}^{z/\sqrt{(4\alpha_h t)}} e^{-\eta^2} d\eta + \frac{2}{\sqrt{\pi}} \int_0^t \frac{d\theta_w}{d\tau} \times \int_{-\infty}^{z/\sqrt{[4\alpha_h(t-\tau)]}} e^{-\eta^2} d\eta \, d\tau. \tag{A.4}$$

As for the temperature of liquid in the film, defining θ_1 and θ_2 to satisfy the following boundary conditions,

$$z = \delta(t): \quad \theta_1 = T_g(t) \qquad \theta_2 = 0$$
$$z = 0 \quad : \quad \theta_1 = 0 \qquad \theta_2 = T_w(t), \tag{A.5}$$

we can rewrite equation (A.1) as

$$\frac{\partial \theta_1}{\partial t} = \alpha_l \frac{\partial^2 \theta_1}{\partial z^2}, \qquad \frac{\partial \theta_2}{\partial t} = \alpha_l \frac{\partial^2 \theta_2}{\partial z^2}, \tag{A.6}$$

of which solutions are

$$\theta_1(z, t) = T_g(0)\left[\frac{z}{\delta} - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \frac{n\pi(\delta - z)}{\delta} \exp(-n^2\pi^2\alpha_l t/\delta)\right] - \int_0^t \frac{dT_g}{d\tau}\left[\frac{z}{\delta} - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \frac{n\pi(\delta - z)}{\delta} \times \exp\{-n^2\pi^2\alpha_l(t - \tau)/\delta\}\right] d\tau \tag{A.7}$$

$$\theta_2(z,t) = T_w(0)\left[\frac{\delta - z}{\delta} - \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin\frac{n\pi z}{\delta}\exp(-n^2\pi^2\alpha_l t/\delta)\right] - \int_0^t \frac{dT_w}{d\tau}\left[\frac{\delta - z}{\delta} - \sum_{n=1}^{\infty}\frac{2}{n\pi}\sin\frac{n\pi z}{\delta} \times \exp\{-n^2\pi^2\alpha_l(t-\tau)/\delta\}\right] d\tau \quad (A.8)$$

where the change in the thickness of the film is neglected compared with that in temperature with respect to time.

The gradients of temperature in the heating material and in the liquid film are given by equations (A.4), (A.7) and (A.8) as follows.

$$\frac{\partial T_h}{\partial z} = \frac{1}{\sqrt{(\pi\alpha_h t)}}\theta_w(0)\exp\left(-\frac{z^2}{4\alpha_h t}\right) + \int_0^t \frac{z^2}{\sqrt{[\pi\alpha_h(t-\tau)]}} \times \frac{d\theta_w}{d\tau}\exp\left[-\frac{z^2}{4\alpha_l(t-\tau)}\right]d\tau \quad (A.9)$$

$$\frac{\partial T_l}{\partial z} = \frac{T_g(0)}{\delta}\left[1 + 2\sum_{n=1}^{\infty}\cos\frac{n\pi(\delta - z)}{\delta}\exp(-n^2\pi^2\alpha_l t/\delta)\right] + \int_0^t \frac{1}{\delta}\frac{dT_g}{d\tau}\left[1 + 2\sum_{n=1}^{\infty}\cos\frac{n\pi(\delta - z)}{\delta} \times \exp\{-n^2\pi^2\alpha_l(t-\tau)/\delta\}\right]d\tau + \frac{T_w(0)}{\delta}\left[1 + 2\sum_{n=1}^{\infty}\cos\frac{n\pi z}{\delta}\exp(-n^2\pi^2\alpha_l t/\delta)\right] + \int_0^t \frac{1}{\delta}\frac{dT_w}{d\tau}\left[1 + 2\sum_{n=1}^{\infty}\cos\frac{n\pi z}{\delta}\exp\{-n^2\pi^2\alpha_l(t-\tau)/\delta\}\right] d\tau. \quad (A.10)$$

Upon using the theta functions

$$\vartheta_0(x,t) = 1 + 2\sum_{n=1}^{\infty}(-1)^n \exp(in^2\pi t)\cos 2n\pi x$$

$$\vartheta_3(x,t) = 1 + 2\sum_{n=1}^{\infty}\exp(in^2\pi t)\cos 2n\pi x,$$

equation (A.10) can be rewritten as

$$\frac{\partial T_l}{\partial z} = \frac{1}{\delta}\left[\vartheta_0\left(\frac{z}{2\delta}, i\pi\frac{\alpha_l t}{\delta^2}\right)T_g(0) + \int_0^t \frac{dT_g}{\delta\tau}\vartheta_0\left(\frac{z}{2\delta}, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)d\tau - \vartheta_3\left(\frac{z}{2\delta}, i\pi\frac{\alpha_l t}{\delta^2}\right)T_w(0) - \int_0^t \frac{dT_w}{d\tau}\vartheta_3\left(\frac{z}{2\delta}, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)d\tau\right], \quad (A.11)$$

which yields the temperature gradient at the surface of the heater and at the film surface,

$$\left(\frac{\partial T_l}{\partial z}\right)_0 = \frac{1}{\delta}\left[\vartheta_0\left(0, i\pi\frac{\alpha\ t}{\delta^2}\right)T_g(0) - \vartheta_3\left(0, i\pi\frac{\alpha_l t}{\delta^2}\right)T_w(0) + \int_0^t \left\{\vartheta_0\left(0, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)\frac{dT_g}{d\tau} - \vartheta_3\left(0, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right) \times \frac{dT_w}{d\tau}\right\}d\tau\right] \quad (A.12)$$

$$\left(\frac{\partial T_l}{\partial z}\right)_\delta = \frac{1}{\delta}\left[\vartheta_3\left(0, i\pi\frac{\alpha_l t}{\delta}\right)T_g(0) - \vartheta_0\left(0, i\pi\frac{\alpha_l t}{\delta}\right)T_w(0) + \int_0^t \left\{\vartheta_3\left(0, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)\frac{dT_g}{d\tau} - \vartheta_0\left(0, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right) \times \frac{dT_w}{d\tau}\right\}d\tau\right]. \quad (A.13)$$

Substitution of equations (A.9) and (A.12) into the continuity relation of heat flux at the heater surface ($z = 0$)

$$\lambda_h\left(\frac{\partial T_h}{\partial z}\right)_0 = \lambda_l\left(\frac{\partial T_l}{\partial z}\right)_0 \quad (A.14)$$

leads to

$$\frac{\lambda_h}{\sqrt{(\pi\alpha_h)}}\left[\frac{1}{\sqrt{t}}(T_w(0) - T_H) + \int_0^t \frac{1}{\sqrt{(t-\tau)}}\frac{d(T_w - T_H)}{d\tau}d\tau\right] = \frac{\lambda_l}{\delta}\left[\vartheta_0\left(0, i\pi\frac{\alpha_l t}{\delta^2}\right)T_g(0) - \vartheta_3\left(0, i\pi\frac{\alpha_l t}{\delta^2}\right)T_w(0) + \int_0^t \left\{\vartheta_0\left(0, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)\frac{dT_g}{d\tau} - \vartheta_3\left(0, i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right) \times \frac{dT_w}{d\tau}\right\}d\tau\right], \quad (A.15)$$

from which the surface temperature of the heater $T_w(t)$ is obtainable. Equation (A.13) with this value of T_w gives the temperature gradient at the film surface, that is, the heat flow through the liquid–vapor interface of the liquid film q_b. To make more perspective insight into the heat-flux, we consider the process of bubble growth in the initial stage ($\alpha_l t/\delta^2 \ll 1$) and in the final ($\alpha_l t/\delta^2 \gg 1$).

For $\alpha_l t/\delta^2 \ll 1$, using Jaccobi's imaginary transformation

$$\vartheta_n(x,t) = \frac{1}{\sqrt{t}}\vartheta_n\left(\frac{x}{t}, -\frac{1}{\sqrt{t}}\right)\exp\left\{i\pi\left(\frac{1}{4} - \frac{x^2}{t}\right)\right\} \quad (n = 0, 3),$$

we can obtain

$$\vartheta_0\left(\frac{z}{2\delta}, i\pi\frac{\alpha_l t}{\delta^2}\right) \approx \vartheta_3\left(\frac{z}{2\delta}, i\pi\frac{\alpha_l t}{\delta^2}\right) \approx \frac{\delta}{\sqrt{(\pi\alpha_l t)}}\exp\left(-\frac{z^2}{4\alpha_l t}\right),$$

which reduces equation (A.11) to be of the same form as equation (A.9). When we represent the continuity relation of heat flux at the heater surface in the sense of

$$\lim_{z\to 0-}\left(\lambda_h\frac{\partial T_h}{\partial z}\right)=\lim_{z\to 0+}\left(\lambda_l\frac{\partial T_l}{\partial z}\right)$$

equation (A.15) can be rewritten as

$$\frac{\lambda_h}{\sqrt{\alpha_h}}\int_0^t\frac{T_w-T_H}{(t-\tau)^{\frac{1}{2}}}\mathrm{d}\tau=\frac{\lambda_l}{\sqrt{\alpha_l}}\int_0^t\frac{T_g-T_w}{(t-\tau)^{\frac{1}{2}}}\mathrm{d}\tau, \quad (A.16)$$

where

$$(t-\tau)^{\frac{1}{2}}\equiv\lim_{z\to 0}(t-\tau)^{\frac{1}{2}}\exp\left(-\frac{z^2}{4\alpha_l(t-\tau)}\right).$$

Thus, we obtain

$$T_w=T_g+\frac{T_H-T_g}{1+\frac{\lambda_l}{\lambda_h}\sqrt{\left(\frac{\alpha_h}{\alpha_l}\right)}}. \quad (A.17)$$

For $\alpha_l t/\delta^2 \gg 1$,

$$\vartheta_0\left(0,i\pi\frac{\alpha_l t}{\delta^2}\right)\approx\vartheta_3\left(0,i\pi\frac{\alpha_l t}{\delta^2}\right)\approx 1$$

$$\vartheta_0\left(0,i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)\approx\vartheta_3\left(0,i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)\approx\begin{cases}\frac{\delta}{\sqrt{[\pi\alpha_l(t-\tau)]}} & (\tau\to t)\\ 1 & (\tau\to 0).\end{cases}$$

Since $\mathrm{d}T_g/\mathrm{d}t$ and $\mathrm{d}T_w/\mathrm{d}t$ are monotonously decreasing functions of time, for sufficiently larger values of time,

$$\int_0^t\vartheta_3\left(0,i\pi\frac{\alpha_l(t-\tau)}{\delta^2}\right)\frac{\mathrm{d}T_w}{\mathrm{d}\tau}\mathrm{d}\tau\approx\int_0^t\frac{\mathrm{d}T_w}{\mathrm{d}\tau}\mathrm{d}\tau=T_w(t)-T_w(0)$$

and

$$\int_0^t\frac{1}{\sqrt{(t-\tau)}}\frac{\mathrm{d}T_w}{\mathrm{d}\tau}\mathrm{d}\tau\approx\frac{1}{\sqrt{t}}\int_0^t\frac{\mathrm{d}T_w}{\mathrm{d}\tau}\mathrm{d}\tau=\frac{T_w(t)-T_w(0)}{\sqrt{t}}.$$

Then, equation (A.15) becomes

$$\frac{\lambda_h}{\sqrt{(\pi\alpha_h t)}}(T_w-T_H)=\frac{\lambda_h}{\delta}(T_g-T_w),$$

which yields the surface temperature of the heater

$$T_w=T_g+\frac{T_H-T_g}{1+\frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_h t)}}{\delta}}. \quad (A.18)$$

For further larger values of time,

$$T_w=T_g+(T_H-T_g)\frac{\lambda_h}{\lambda_l}\frac{\delta}{\sqrt{(\pi\alpha_h t)}}.$$

The same manipulation gives the temperature gradient at the film surface

$$\left(\frac{\partial T_l}{\partial z}\right)_\delta=\begin{cases}\frac{1}{\sqrt{(4\pi\alpha_l)}}\int_0^t\frac{T_w-T_g}{(t-\tau)^{\frac{3}{2}}}\mathrm{d}\tau & \frac{\alpha_l t}{\delta^2}\ll 1\\ \frac{T_g-T_w}{\delta} & \frac{\alpha_l t}{\delta^2}\gg 1,\end{cases} \quad (A.19)$$

which are rewritten by equations (A.17) and (A.18) as

$$\left(\frac{\partial T_l}{\partial z}\right)_\delta=\begin{cases}\frac{1}{\sqrt{(4\pi\alpha_l)}}\left\{1+\frac{\lambda_l}{\lambda_h}\sqrt{\left(\frac{\alpha_h}{\alpha_l}\right)}\right\}^{-1}\int_0^t\frac{T_H-T_g}{(t-\tau)^{\frac{3}{2}}}\mathrm{d}\tau & \frac{\alpha_l t}{\delta^2}\ll 1\\ \frac{1}{\delta}\left(1+\frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_l t)}}{\delta}\right)^{-1}(T_g-T_H) & \frac{\alpha_l t}{\delta^2}\gg 1.\end{cases} \quad (A.20)$$

Accordingly, we obtain the heat flow through the liquid film

$$q_b=\begin{cases}\int_{S_b}\frac{\lambda_l}{\sqrt{(4\pi\alpha_l)}}\left\{1+\frac{\lambda_l}{\lambda_h}\left(\frac{\alpha_h}{\alpha_l}\right)\right\}^{-1}\int_0^t\frac{T_g-T_H}{(t-\tau)^{\frac{3}{2}}}\mathrm{d}\tau\,\mathrm{d}S_b & \frac{\alpha_l t}{\delta^2}\ll 1\\ \int_{S_b}\frac{\lambda_l}{\delta}\left(1+\frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_h t)}}{\delta}\right)^{-1}(T_H-T_g)\,\mathrm{d}S_b & \frac{\alpha_l t}{\delta^2}\gg 1.\end{cases} \quad (A.21)$$

Next, we consider the heat flux through the liquid–vapor interface except that of the liquid film. Though, as to the temperature field of liquid surrounding a spherical bubble, Plesset, Forster *et al.* conducted detailed studies, here we treat the temperature field in the vicinity of growing bubbles in a following brief manner. Introducing the co-ordinate y with the origin at the interface directing towards the liquid side and taking the assumption of local spherical symmetry, we obtain the energy equation of liquid

$$\frac{\partial T_l}{\partial t}-\dot{R}_0\frac{\partial T_l}{\partial y}+v\frac{\partial T_l}{\partial y}=\frac{\alpha_l}{(R_0+y)^2}\frac{\partial}{\partial y}\left[(R_0+y)^2\frac{\partial T_l}{\partial y}\right] \quad (A.22)$$

where $\dot{R}_0$ is the moving speed of the interface and v is the y-component of velocity which is roughly proportional to

$\dot{R}_0[R_0/(R_0 + y)]^2$. For the region of $y \ll R_0$, the above equation can be approximated by

$$\frac{\partial T_l}{\partial l} = \alpha_l \frac{\partial^2 T_l}{\partial y^2}, \qquad \text{(A.22)}$$

for which the boundary conditions are expressed as

$$y = \infty: \quad T_l(\infty, t) = T_L$$
$$y = 0: \quad T_l(0, t) = T_g,$$

where T_L is the temperature of liquid at infinity $y = \infty$. The thickness of temperature boundary layer in the surrounding liquid is about $\sqrt{(\alpha_l t)}$, that is, of the order of micron, while the boundary layer of temperature immediately before the bubble ebullition is of the thickness of millimeter, so that T_L can be regarded to be weakly dependent on time. Setting

$$\theta = T_l - T_L \qquad \text{(A.23)}$$

and solving the above equation, we obtain

$$\theta(y, t) = \frac{2}{\sqrt{\pi}} \int\limits_{y/\sqrt{4\alpha_l t}}^{\infty} e^{-\eta^2} \mathrm{d}\eta \, \theta_g(0) + \frac{2}{\sqrt{\pi}} \int\limits_0^t \frac{\mathrm{d}\theta_g}{\mathrm{d}\tau} \int\limits_{y/\sqrt{[4\alpha_l(t-\tau)]}}^{\infty} e^{-\eta^2} \mathrm{d}\eta \, \mathrm{d}\tau, \qquad \text{(A.24)}$$

where $\theta_g = T_g - T_L$. The temperature gradient is then

$$\frac{\partial T_l}{\partial y} = \frac{1}{\sqrt{(4\pi\alpha_l)}} \int\limits_0^t \left(1 - \frac{y^2}{4\alpha_l(t-\tau)}\right) \frac{1}{(t-\tau)^{3/2}} \times \exp\left(-\frac{y^2}{4\alpha_l(t-\tau)}\right)(T_g - T_L)\,\mathrm{d}\tau. \qquad \text{(A.25)}$$

Thus, the heat flow through the interface q_a becomes

$$q_a = \int\limits_{S_a} \frac{\lambda_l}{\sqrt{(4\pi\alpha_l)}} \int\limits_0^t \frac{T_g - T_L}{(t-\tau)^{1/2}} \mathrm{d}\tau \, \mathrm{d}S_a. \qquad \text{(A.26)}$$

For larger values of t, if the change in $(T_g - T_L)$ can be neglected compared with that in $(t - \tau)^{1/2}$, the same manipulation as for equation (A.18) yields

$$q_a = \int\limits_{S_a} \frac{\lambda_l}{\sqrt{(\pi\alpha_l t)}} (T_L - T_g)\,\mathrm{d}S_a. \qquad \text{(A.27)}$$

If the uniformness of the quantities at the interface is assumed, the above obtained results of heat flow are rewritten as

$$q_a = \begin{cases} \dfrac{\lambda_l}{\sqrt{(4\pi\alpha_l)}} S_a \displaystyle\int\limits_0^t \frac{T_g - T_L}{(t-\tau)^{1/2}} \mathrm{d}\tau & \dfrac{\alpha_l t}{\delta^2} \ll 1 \\ \dfrac{\lambda_l}{\sqrt{(\pi\alpha_l t)}} S_a (T_L - T_g) & \dfrac{\alpha_l t}{\delta^2} \gg 1 \end{cases} \qquad \text{(2.4)}$$

$$q_b = \begin{cases} \dfrac{\lambda_l}{\sqrt{(4\pi\alpha_l)}} \left[1 + \dfrac{\lambda_l}{\lambda_h}\sqrt{\left(\dfrac{\alpha_h}{\alpha_l}\right)}\right]^{-1} S_b \displaystyle\int\limits_0^t \frac{T_g - T_H}{(t-\tau)^{1/2}} \mathrm{d}\tau & \dfrac{\alpha_l t}{\delta^2} \ll 1 \\ \displaystyle\int\limits_{S_b} \frac{\lambda_l}{\delta}\left(1 + \frac{\lambda_l}{\lambda_h}\frac{\sqrt{(\pi\alpha_h t)}}{\delta}\right)^{-1} \mathrm{d}S_b (T_H - T_g) & \dfrac{\alpha_l t}{\delta^2} \gg 1, \end{cases} \qquad \text{(2.5)}$$

where $(t - \tau)^{1/2}$ has the meaning defined previously.

FILM LIQUIDE AU COURS D'UNE ÉBULLITION NUCLÉÉE

Résumé—En examinant près de la surface chauffante, le mouvement du liquide entourant une bulle et accompagnant la croissance de celle-ci, on en conclut que durant l'ébullition nucléée une fine couche liquide est formée thermiquement et dynamiquement à la base de la bulle sur la surface chauffante. Cette couche a une épaisseur inversement proportionnelle à la racine carrée du nombre de Reynolds, basé sur le rayon et la vitesse de croissance de la bulle, où, au produit du rayon de la bulle et du rapport de densité de la vapeur et du liquide.

Au premier état de croissance de la bulle, le film liquide influence le processus seulement par la croissance corrélative de l'aire de l'interface liquide-vapeur. A un stade plus avancé, une bulle avec un film liquide croît à la même vitesse que celle sans film, bien que la surchauffe du liquide ait moins d'influence sur cette dernière.

Avec un film plus fin, la vitesse de croissance est affectée par les propriétés thermiques du chauffoir, étant trop élevée pour être réalisé lors de l'ébullition nucléée usuelle de l'eau.

ÜBER DEN FLÜSSIGKEITSFILM BEIM BLASENSIEDEN

Zusammenfassung—Auf Grund der Untersuchung der Flüssigkeitsbewegung in der Umgebung einer Blase dicht an der Heizfläche wird gezeigt, dass sich beim Blasensieden flüssigkeitsdynamisch oder thermisch eine dünne Flüssigkeitsschicht an der Blasenunterseite auf der Heizfläche bildet. Die

Schichtdicke ist umgekehrt proportional zur Quadratwurzel aus der Reynolds-Zahl, gebildet mit dem Blasenradius und seiner Wachstumsgeschwindigkeit oder zum Blasenradius mal dem Dichteverhältnis von Dampf zu Flüssigkeit.

Im Anfangsstadium des Blasenwachsens hat der Flüssigkeitsfilm nur durch die damit verbundene Zunahme der Dampf-Flüssigkeits-Grenzfläche Einfluss auf den Prozess. In fortgeschrittenem Stadium wächst eine Blase mit Flüssigkeitsfilm mit ähnlicher Geschwindigkeit wie eine ohne Film, obwohl die Flüssigkeitsüberhitzung weniger Einfluss hat. Bei wesentlich dünnerem Film ist die Wachstumsgeschwindigkeit von thermischen Eigenschaften der Heizung bestimmt; sie ist dann zu gross, um sich bei normalem Blasensieden von Wasser realisieren zu lassen.

О ЖИДКОЙ ПЛЕНКЕ ПРИ ПУЗЫРЬКОВОМ КИПЕНИИ

Аннотация—Исследуется движение жидкости вокруг пузыря, образующегося вблизи поверхности нагрева, сопровождающего вскипание и рост пузыря, при этом доказывается, что образование тонкого слоя жидкости у основания пузыря на поверхности нагрева при пузырьковом кипении имеет гидродинамический или тепловой характер, что толщина слоя обратно пропорциональна корню квадратному числа Рейнольдса, отнесенного к радиусу пузыря и скорости его роста или к радиусу пузыря, умноженному на отношение плотности пара к плотности жидкости.

В самом начале роста пузыря пленка жидкости влияет на процесс только через увеличение поверхности раздела жидкость — пар. В дальнейшем пузырь с пленкой жидкости растет со скоростью, близкой к скорости роста при отсутствии пленки, хотя перегрев жидкости оказывает меньшее влияние в первом случае. При гораздо более тонких пленках на скорость роста влияют тепловые свойства нагревателя, причем эта скорость настолько велика, чтобы это влияние можно было заметить при обычном пузырьковом кипении воды.